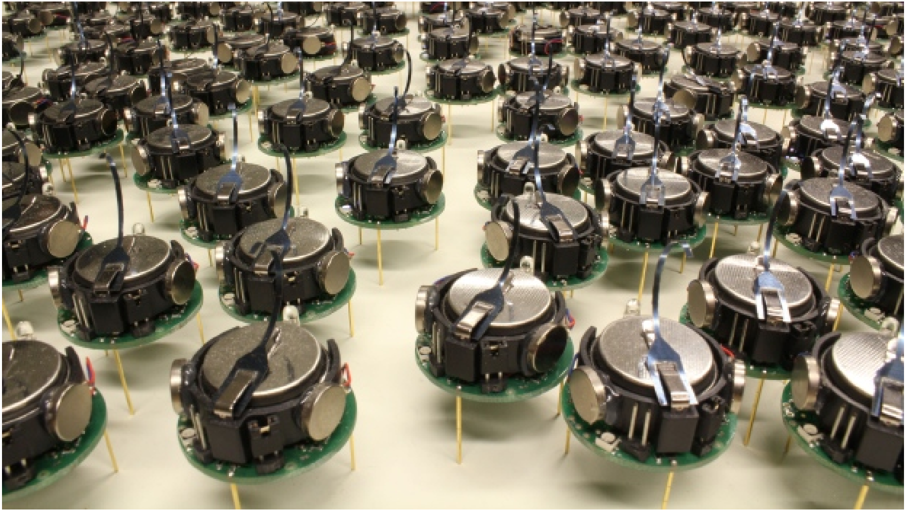
Swarms of Mobile Robots

the Quest for Safety

X. Urbain

Autonomous Mobile Robots



AUTONOMOUS Mobile Robots

Autonomous Mobile Robots

Autonomous Mobile Robots

Autonomous Mobile Robots

Robotic Swarms cooperate to tasks

- Monitoring (wildlife...)
- Ad-hoc networks
- Exploration
- Patrol
- Search and rescue
- ...

Environment $\rightsquigarrow$ harmful ?

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SCIENCE \ US & WORLD \ TECH

Fukushima's record-high radiation broke a cleaning robot after two hours

Radiation levels are clocking in at 650 Sieverts per hour

by Natt Garun | @nattgarun | Feb 10, 2017, 4:16pm EST

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A robot sent into a Fukushima reactor to inspect and clean the nuclear plant had to abruptly end its mission after excess radiation fried the robot's camera. It was the first time a robot had entered the Unit 2 reactor since the 2011 earthquake and tsunami, reports the *Associated Press*.

NOW TRENDING

7 things we learned from Elon Musk's Tesla shareholder meeting

HP gets in on the external GPU hype with a pretty, large box

Robotic Swarms cooperate to task

- Ad-hoc networks
- Exploration
- Patrol
- Search and rescue
- …

Environment $\rightsquigarrow$ harmful ?

- Low cost — the user can afford to lose some

Robotic Swarms cooperate to task

- Ad-hoc networks
- Exploration
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- Search and rescue
- ...

Environment $\rightsquigarrow$ harmful ?

- Low cost — the user can afford to lose some
- Robust protocols — the task can afford to lose some

$\rightsquigarrow$ Weak assumptions — capabilities, silence...

... and **uniform way to describe them**

Robotic Swarms   cooperate to task

- Ad-hoc networks
- Exploration
- Patrol
- Search and rescue
- ...

Lives $\leadsto$ critical !

- Verification/certification

$\leadsto$ **Formal methods**

Certification

Subtle differences + informal reasoning $\leadsto$ error prone

Protocols incorrect w.r.t. specifications still found (see EDCC'15)

Model recent, developping + critical app. $\leadsto$ need formally verified ground

Formal methods

- Model-checking : reachability LTL [Bérard. . ., Devisme. . .]
  + automated – instances (small, discrete)
- Synthesis [Bonnet. . ., Millet. . .]
- Formal proof : proof assistant Coq, Isabelle
  – expertise + scalable, general

$\leadsto$ 2 phases spec/proof : emphasis on ease of specification

Robotic Swarms the need

From a computer science perspective

- Clear characterization, computation model ($\rightsquigarrow$ "realistic")
- Means of verification (matching variants)

$\rightsquigarrow$ Suzuki & Yamashita's model + formal framework

$\rightsquigarrow$ Pactole formal library Courtieu/Rieg/Tixeuil/Urbain

Autonomous Mobile Robots

Suzuki & Yamashita 1999

Robots Move in **Space** according to their **Perception**

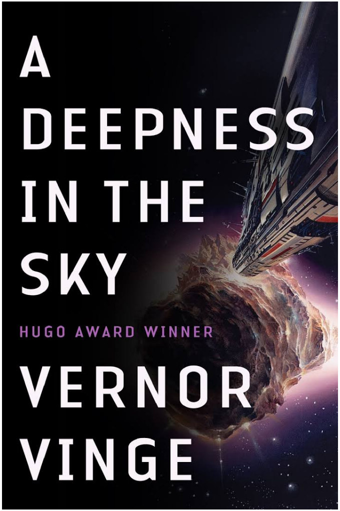
Following :  cycle $\triangle$

Autonomous agents $\rightsquigarrow$ cooperation to realise a task

Without any explicit communication channel

$\rightsquigarrow$ many variants. . .

Autonomous Mobile Robots . . . also in 1999



Spoiler : robotic swarms save the day !

Autonomous Mobile Robots

Suzuki & Yamashita 1999

Robots Move in **Space** according to their **Perception**

Following :  cycle △

Autonomous agents ⇝ cooperation to realise a task

Without any explicit communication channel

⇝ many variants. . . ⇝ "realistic" assumptions

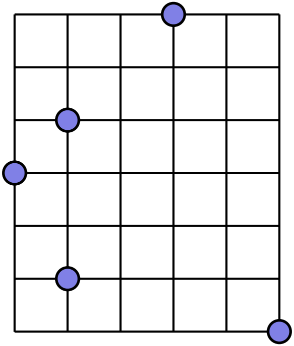
- Space/topology
- Robots' structure and capabilities
- Synchronisation model

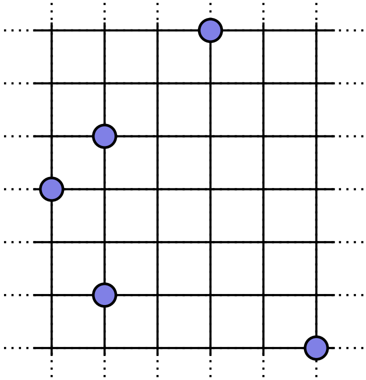
Suzuki & Yamashita 1999

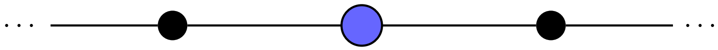
Robots Move in **Space** according to their **Perception**

Characteristics of space

- Topology ?
- Size ?
- . . .

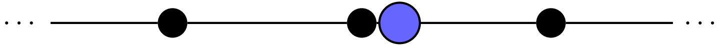


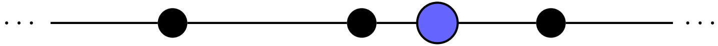


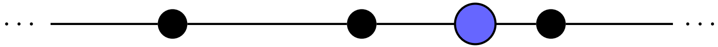


Autonomous Mobile Robots

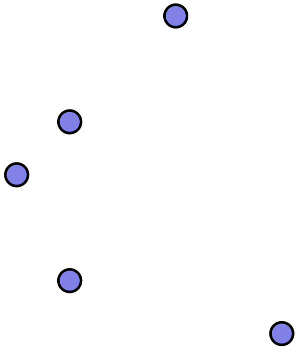
continuous **space**







Autonomous Mobile Robots — continuous **space**



Formalisation **space**

Need for various topologies + generic operations

$\rightsquigarrow$ Space : parameter

Defined as Module `Loc`

- Core : `Loc.t` position in space
- Utils : origin
- Utils : decidable equality
- Utils : distance
- Utils : ...

Instances : graph, $\mathbb{Z}/n\mathbb{Z}$, $\mathbb{R}$, $\mathbb{R}^n$ with relevant arithmetics... that's it

Autonomous Mobile Robots

Suzuki & Yamashita 1999

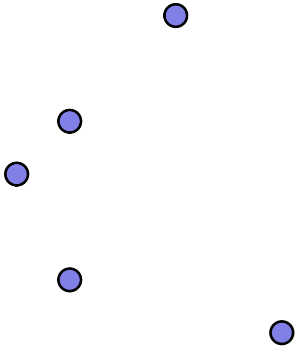
Robots **Move** in **Space** according to their **Perception**

Capabilities of robots

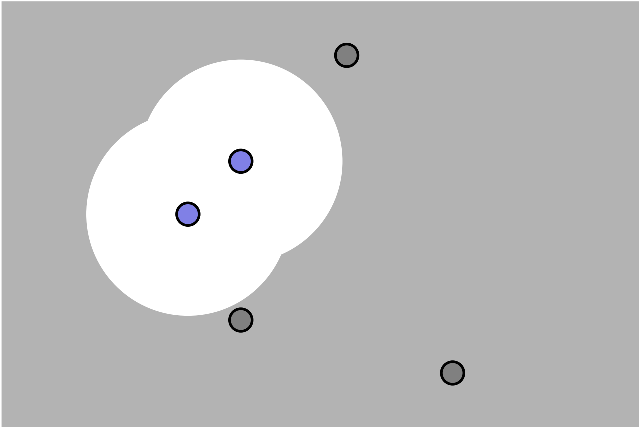
- Memory/Oblivious ?
- Limited/Unlimited perception ?
- What is perceived ? names ? multiplicities ? (“3 here” $\neq$ “Some here”)
- Shared orientation/chirality ?
- Volume ? Energy ? no collision no obstruction ?
- …

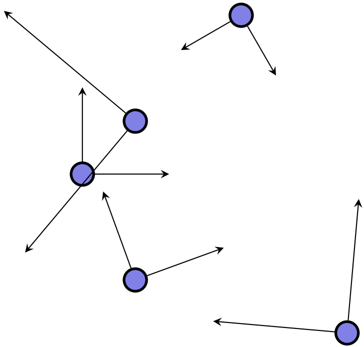
Autonomous Mobile Robots

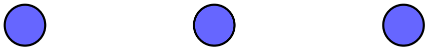
unlimited **vision**



limited **vision**



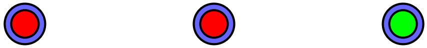




Autonomous Mobile Robots colours

Autonomous Mobile Robots

colours



Autonomous Mobile Robots

Self-visible ?

Formalisation robots, configurations

Robots : set of id., $R = G \uplus B$ size $N = nG + nB$

Conformation : internal state

```
Record Info : Type := (* some state description *) .
Record RobotConf := { loc :> Loc ; robot_info: Info } .
```

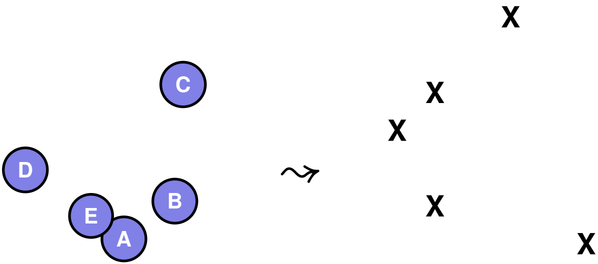
Configuration : display of any robot's conformation simply function

```
Definition configuration := identifier → RobotConf.
```

module with utils : equivalences of config…

Usually too accurate : not the same as perception

Formalisation

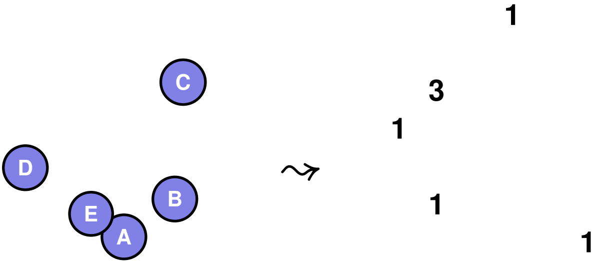


- Roughness of information : observations
- Again a module : equivalence, `from_config`, adequateness. . .

Core : simply function over config → (multi-)sets. . .

Formalisation

perception



- Roughness of information : observations
- Again a module : equivalence, `from_config`, adequateness...

Core : simply function over config → (multi-)sets...

Formalisation embedded program

Algorithm

INPUT : observation wrt r_i

OUTPUT : (conformation with) destination location wrt r_i

Properties

Equivalent perception $\leadsto$ equivalent result

Algorithm + Properties = Robogram

```
Record robogram := {
  pgm :> Obs.t → RobotConf;
  pgm_compat : Proper (Obs.eq ⇒ RobotConf.eq) pgm}.
```

Completely abstract

Higher Order : we can quantify !

Algorithm

INPUT : spectrum wrt r_i

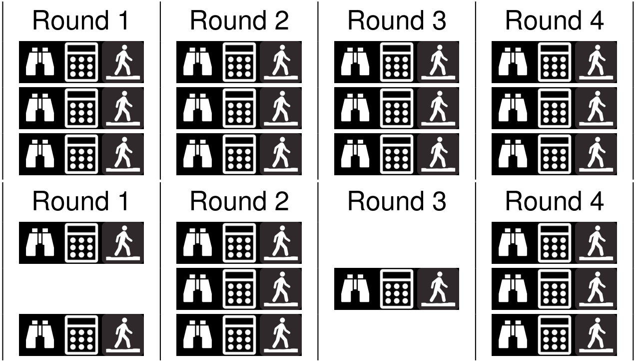
OUTPUT : (conformation with) destination location wrt r_i

```
Definition robogram_pgm (s: Obs.t) : R.t :=
  match Obs.support (Smax s) with (* loc. of max mult. *)
    | nil ⇒ 0                          (* absurd *)
    | pt :: nil ⇒ pt                   (* I: 1 highest tower*)
    | _ ⇒                              (* ... otherwise...  *)
       if beq_nat (length (Obs.support s)) 3 then
       List.nth 1 (sort (Obs.support s)) 0 (*II: mid one *)
       else if is_extremal 0 s then 0    (* ... stay... *)
            else extreme_center s        (* III: center *)
```

Suzuki & Yamashita 1999

Robots **Move** in **Space** according to their **Perception**

Synchronisation and **Movement**



FSYNC

SSYNC

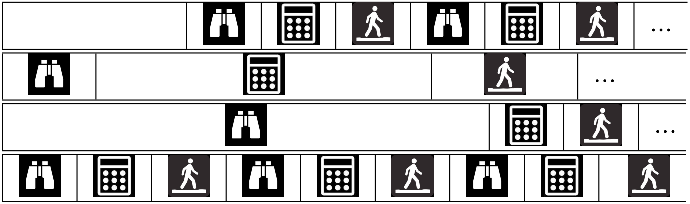
Autonomous Mobile Robots

core

Suzuki & Yamashita 1999

Robots **Move** in **Space** according to their **Perception**

Synchronisation and **Movement**



ASYNC

⇝ outdated perceptions...

$\text{ASYNC} \subseteq \text{SSYNC} \subset \text{ASYNC}^{O(1)}$

Autonomous Mobile Robots core

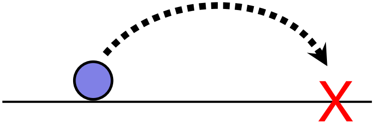
Suzuki & Yamashita 1999

Robots **Move** in **Space** according to their **Perception**

Synchronisation and **Movement**

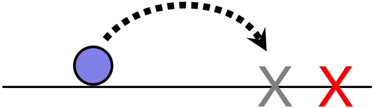
Rigid

X always reached



δ-Flexible

dist $\geq \delta$ if X not reached



Autonomous Mobile Robots

Suzuki & Yamashita 1999

Robots Move in **Space** according to their **Perception**

Core of the model : act of demon (`demonic_action`)

- Selects robots to be activated & assigns changes of state
- Assigns locations/state to Byzantine

F/SSYNC $\rightsquigarrow$ by round, ASYNC $\rightsquigarrow$ by internal state [SSS18 + NETYS19]

Demon

Infinite sequence of its actions : head action, and the rest... (stream)

Round for flexible mvt : new config = new function Proof user only !

```
Definition round δ prot (da: demonic_action) (config) :=
  fun id ⇒
    let loc := config id in (* current loc seen by demon *)
    match da.(step) id with (* is the robot activated? *)
      | None ⇒ loc (* not activated: stay. Never in FSYNC
      | Some (sim, ratio) ⇒ (* new frame, move ratio *)
        let frame_change := sim (config id) in
        let local_config := map frame_change config in
        let local_target := prot (from_config local_config)
        let chosen_target := Loc.mul ratio local_target in
       {| frame_change⁻¹
       (if δ ≤ (Loc.dist (frame_change⁻¹ chosen_target) loc)
           then chosen_target else local_target) ; ... |}
```

Round for flexible mvt : new config = new function Proof user only !

[• • •]

What left for Spec user ?

- Context if not in libraries
- Protocol if needed (correctness)
- Problem / Correctness theorem if not in libraries

Questions :

- What is possible ?
- How it is possible ?
- Is that correct ?

Popular Problems

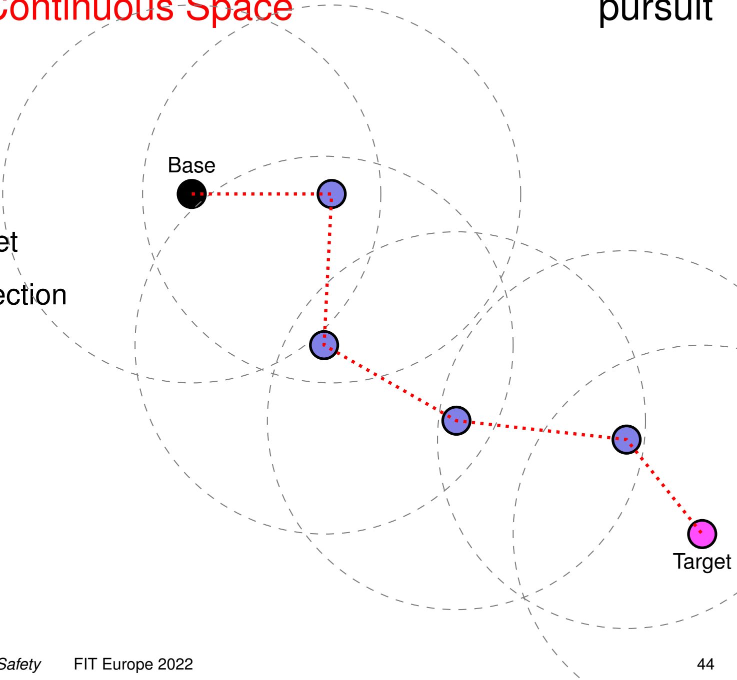
- Probe (patrol/exploration)
- Pursuit (flock/school)
- Pattern formation

Swarms in Continuous Space

Special points :

- Base
- Mobile target

Invariant : connection



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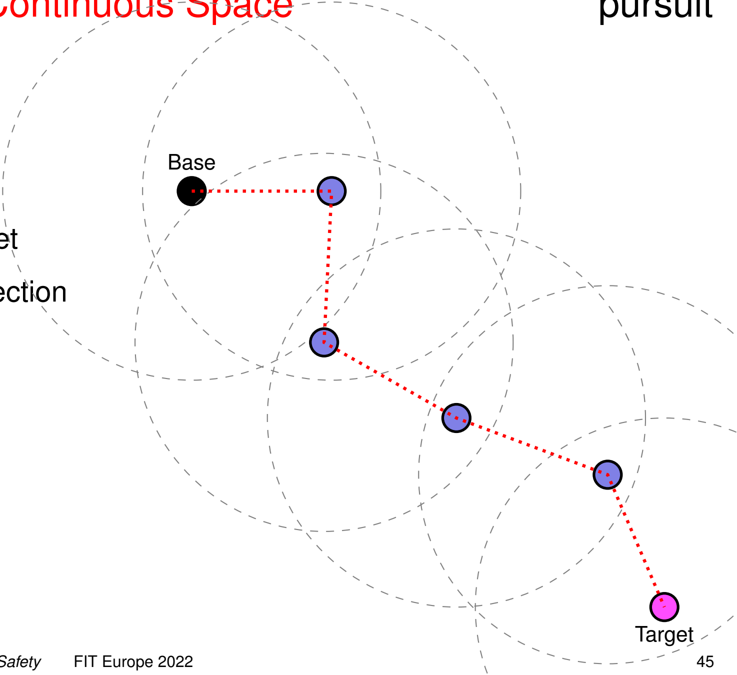

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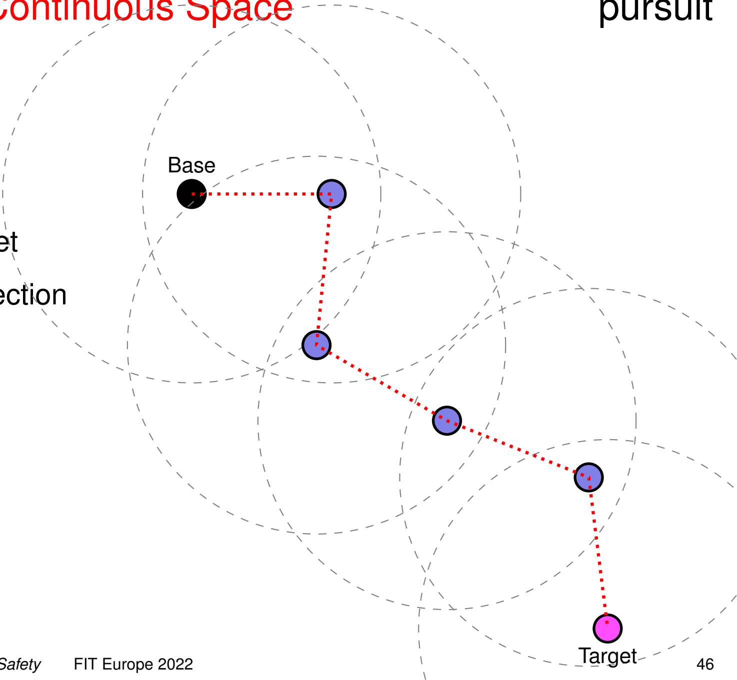

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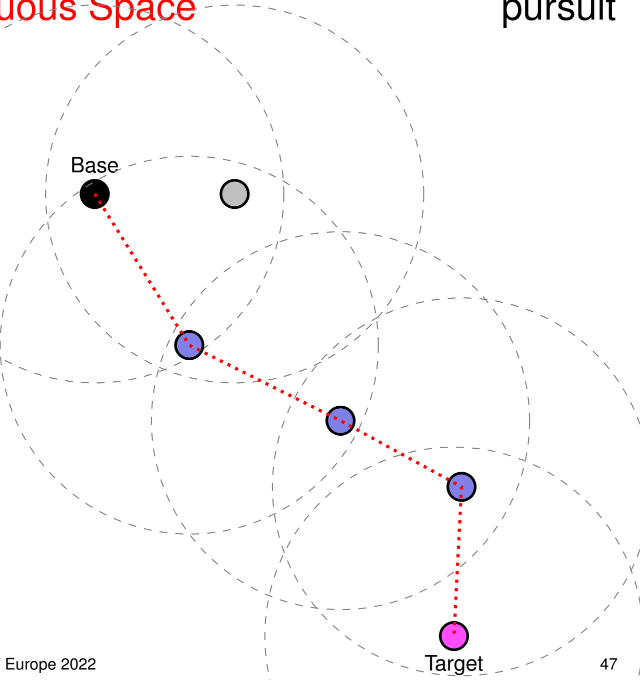
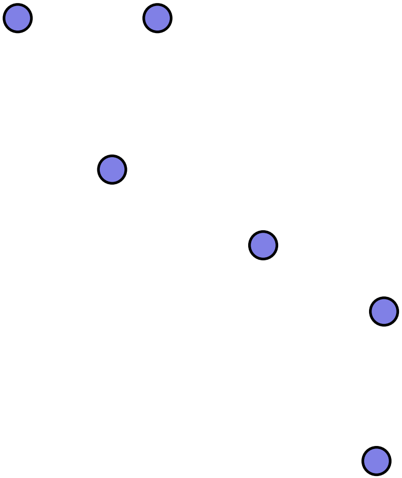


Swarms in Continuous Space

Origin : any conf. (scattered)

Goal : Arbitrary pattern



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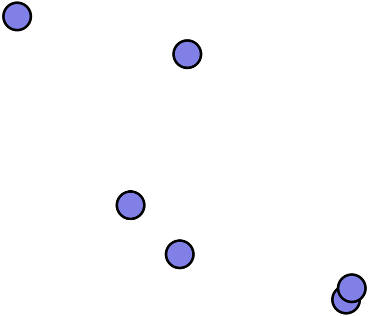

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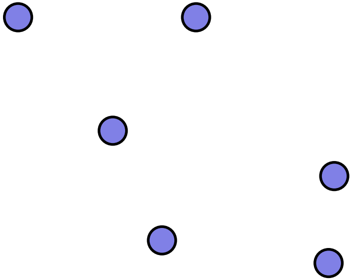

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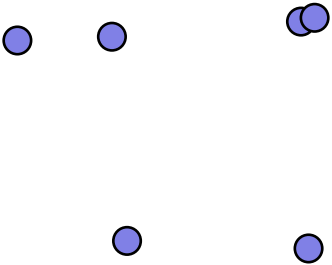

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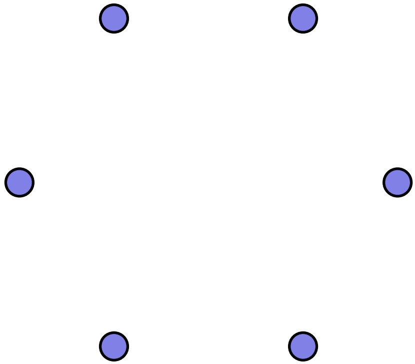

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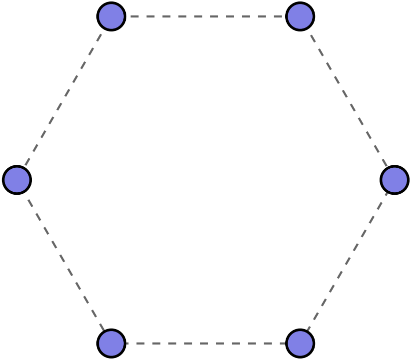

Swarms in Continuous Space

Origin : any conf. (scattered)

Goal : Arbitrary pattern

For $n \geq 3$, APF allows for Leader Election

$\rightsquigarrow$ unsolvable for FSYNC det. oblivious rigid silent robots + chirality [Flocchini et al. 08]

With LE mechanism + chirality $\rightsquigarrow$ solvable in ASYNC [Dieudonné et al. 10]

With full compass $\rightsquigarrow$ solvable in ASYNC [Flocchini et al. 08]

With one axis $\rightsquigarrow$ solvable for odd number in ASYNC

Sequence $\rightsquigarrow$ message...

Swarms in Continuous Space

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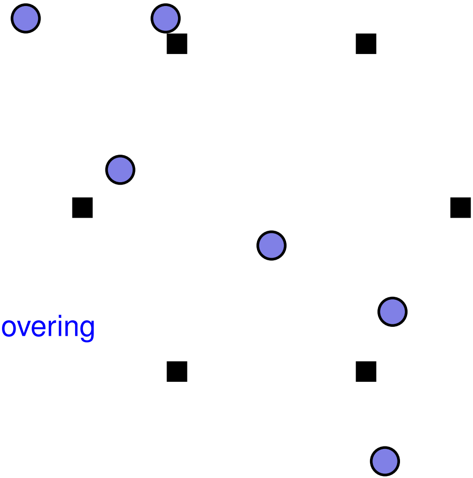

Visible/known $\rightsquigarrow$ Landmark Covering

Swarms in Continuous Space

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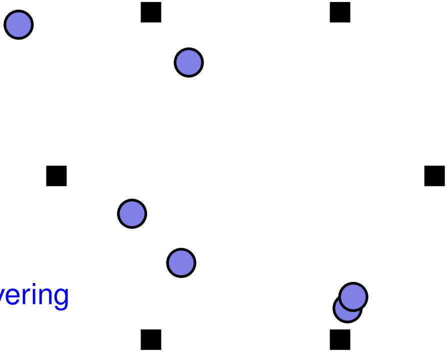

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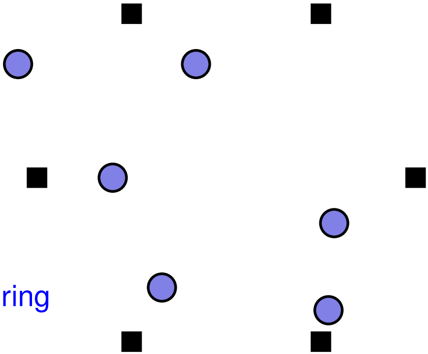

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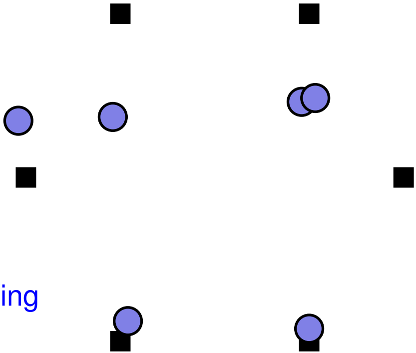

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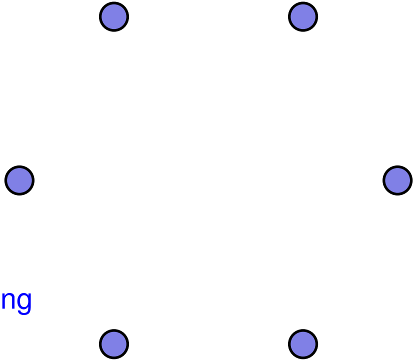

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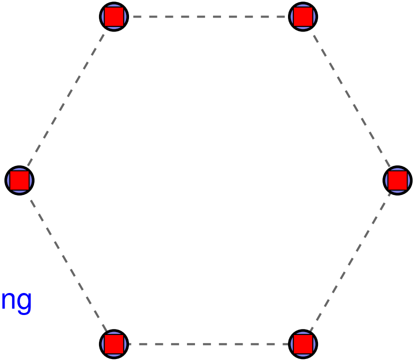

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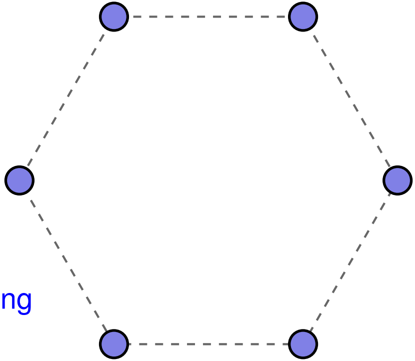


Visible/known $\leadsto$ Landmark Covering

Swarms in Continuous Space pattern

Origin : any conf. (scattered)

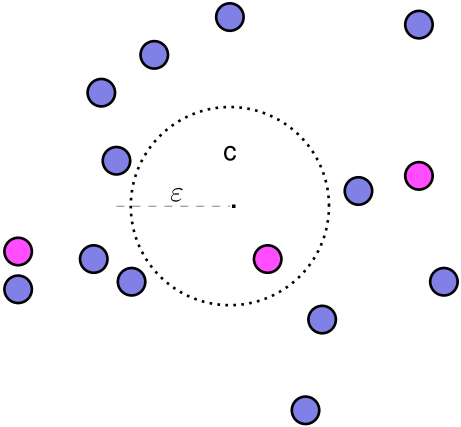
Goal : Arbitrary pattern

With chirality $\rightsquigarrow$ solvable in ASYNC [Fujinata et al. 2010]
“clockwise matching”

Subcases : benchmarks

- Uniform circle/polygons, *regular*-ish shapes…
- Convergence/Gathering

Swarms in Continuous Space



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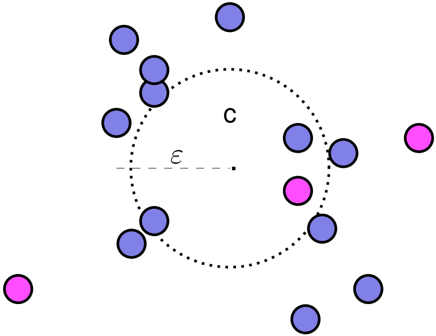

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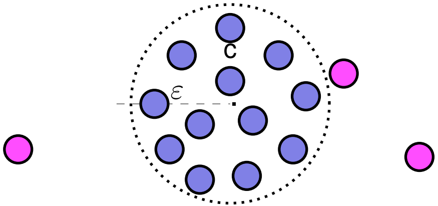
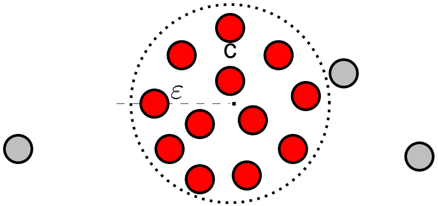
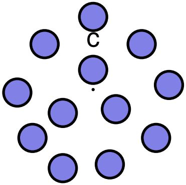




- Forever
  within ε from c
  (coinductive)
- Eventually there
  (inductive)

Convergence :
$\exists c, \forall \varepsilon \ldots$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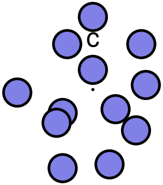
Swarms in Continuous Space



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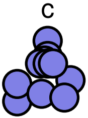
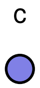


Swarms in Continuous Space

gathering



Swarms in Continuous Space

- Forever at c
  (coinductive)
- Eventually there
  (inductive)

Gathering : $\exists c, \ldots$

Formalisation


```
Definition Gather (pt: Loc.t) (e : execution) :=
Stream.forever (Streams.instant (gathered_at pt)) e.
```

```
Definition WillGather (pt : Loc.t) (e : execution) :=
Stream.eventually (Gather pt) e.
```

Without or with any additional initial condition. . .

```
Definition FullSolGathering (r : robogram) (d : demon) :=
∀ config, ∃ pt : Loc.t, WillGather pt (execute r d config).
```

```
Definition ValidSolGathering (r : robogram) (d : demon) :=
∀ config, ¬invalid config → ∃ pt : Loc.t, WillGather pt...
```

An example

- Anonymous, uniform, points
- Oblivious
- Silent
- No shared orientation

- $\mathbb{R}^2$
- Rigid
- SSYNC

An example

IMPOSSIBILITY of gathering even

Theorem

Gathering impossible for even number of oblivious robots with SSYNC k-fair demon ($k \geq 1$).

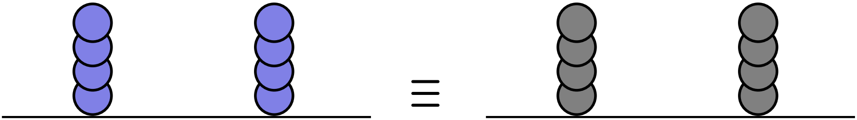
```
Hypothesis even_nG : Nat.Even N.nG.
Hypothesis nG_non_0 : N.nG ≠ 0.

Theorem noGathering:
  ∀ k, (1≤k) → ¬(∀ d, kFair k d → FullSolGathering r d).
```

That's all for specifications... (Proof ~400 lines)

An example

IMPOSSIBILITY of gathering even



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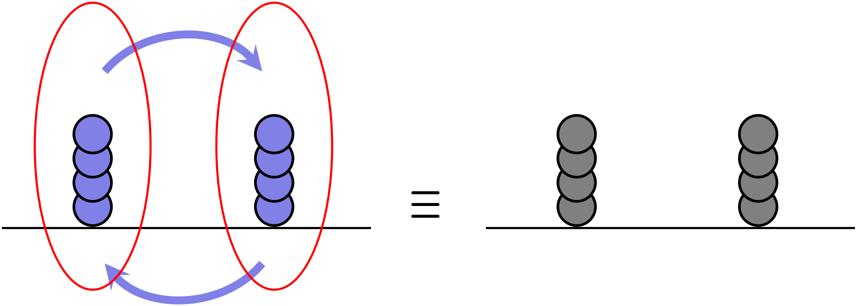

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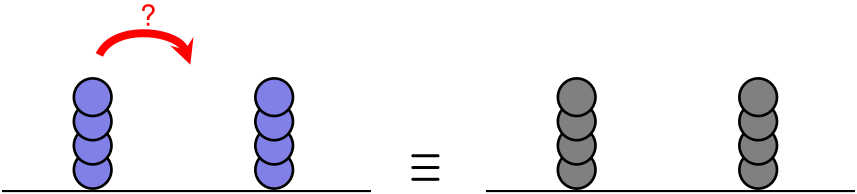

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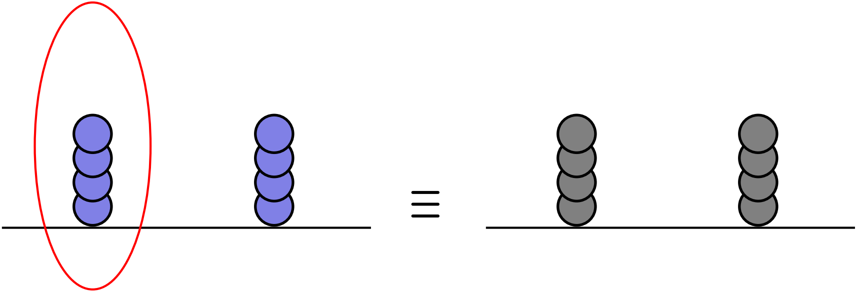

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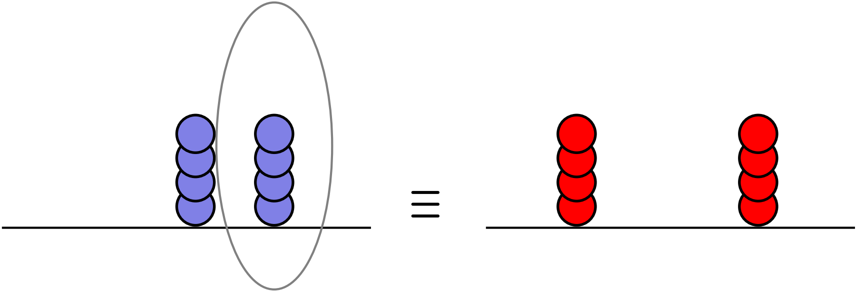

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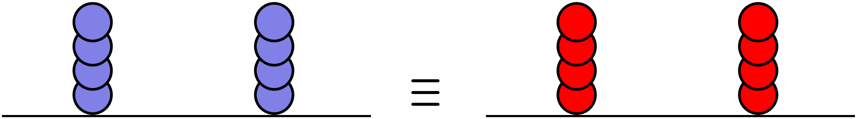

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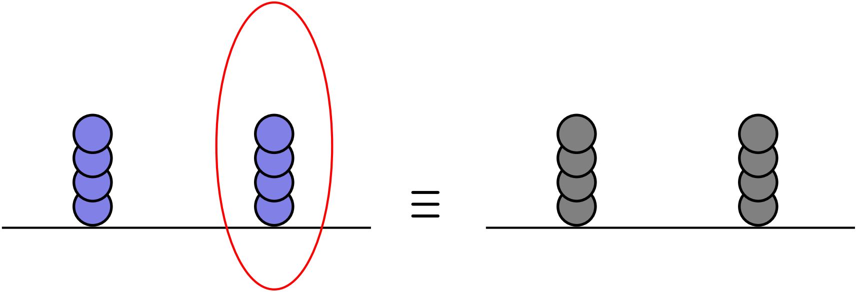

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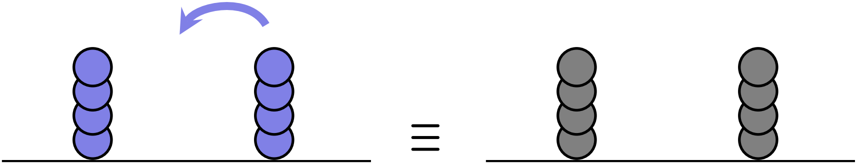

Robotic Swarms the need

From a computer science perspective

- Clear characterization, computation model ($\leadsto$ “realistic”)
- Means of verification (matching variants)

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Fundamental problems Gathering...

Robotic Swarms — the need

From a computer science perspective

- Clear characterization, computation model ($\rightsquigarrow$ “realistic”)
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In practice : *Problem oriented* (bounds, optimality)

Fundamental problems $\rightsquigarrow$Disconnection

Robotic Swarms

the need

From a computer science perspective

- Clear characterization, computation model ($\rightsquigarrow$ “realistic”)
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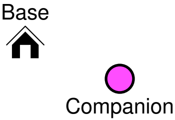
In practice : *Problem oriented* (bounds, optimality)

How to obtain a correct protocol ?

The Life-Line Problem

Given Points :

- Base
- Companion



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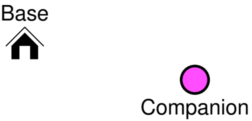

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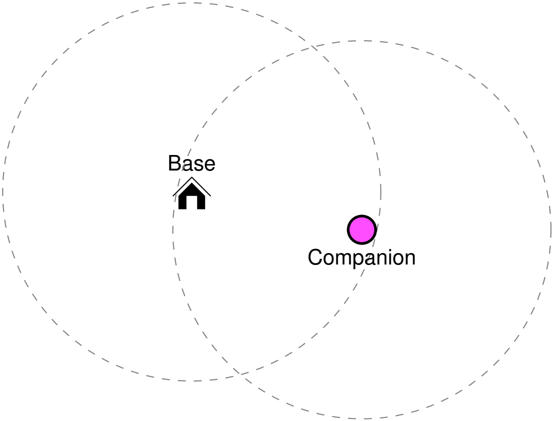

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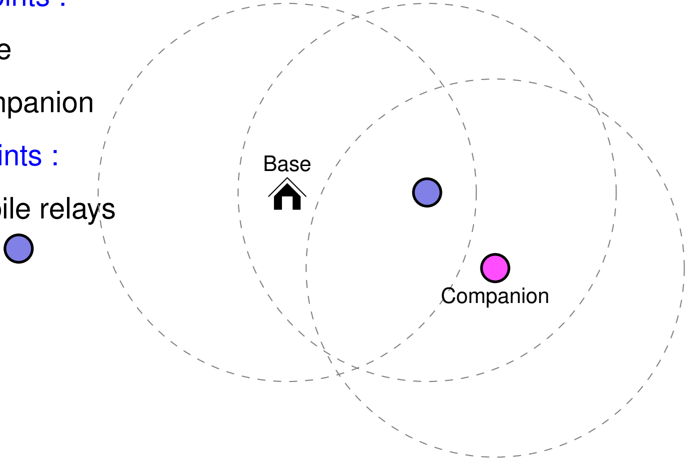
Extra Points :

- mobile relays



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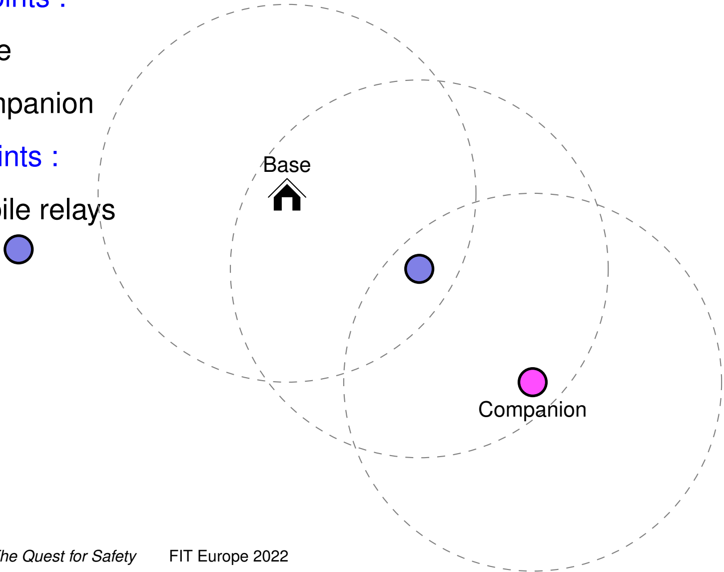

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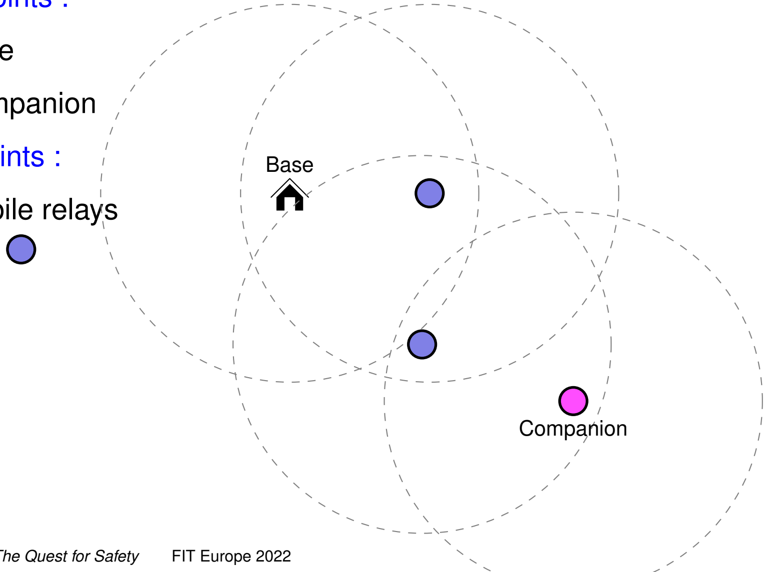

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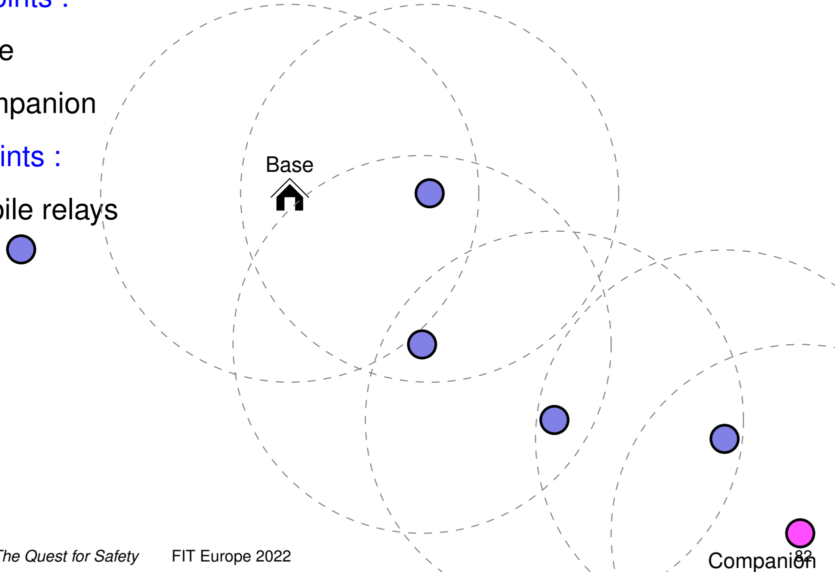

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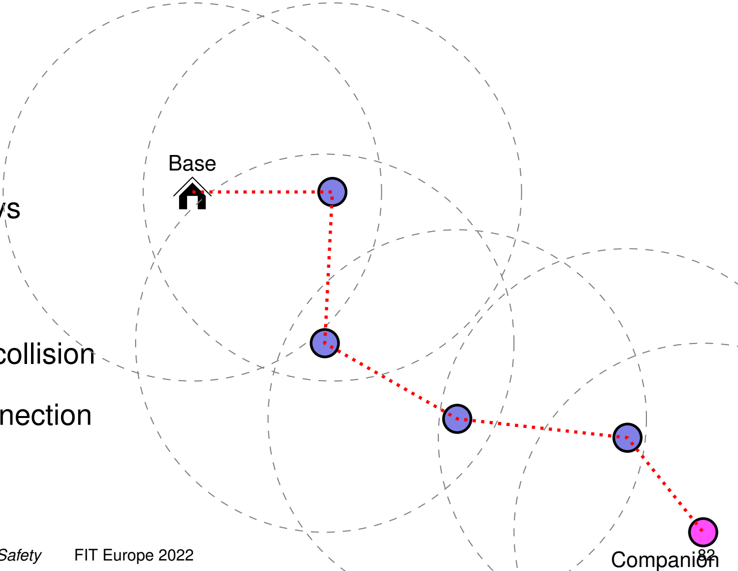

Invariant 1 : no collision

Invariant 2 : connection

The Life-Line Problem

- $\mathbb{R}^3 \rightsquigarrow \mathbb{R}^2$ flying

Companion $\neq$ search team

- Special location : base

The Life-Line Problem

- $\mathbb{R}^3 \rightsquigarrow \mathbb{R}^2$ flying — Companion ≠ search team
- Special location : base

- No detection of multiplicity (volume/collision)
- Vision limitée — D_{max}
- Vitesse limitée
- No Byzantine

The Life-Line Problem

- $\mathbb{R}^3 \rightsquigarrow \mathbb{R}^2$ flying — Companion ≠ search team
- Special location : base
- No detection of multiplicity (volume/collision)
- Vision limitée — D_{max}
- Vitesse limitée
- No Byzantine
- FSYNC
- Rigid mvt
- Start from base

The Life-Line Problem

Invariants formal statement

No collision

```
∀ r r′, r ≠ r′ →location(r) ≠ location(r′)
```

Connected path

```
∀ r, alive(r) →
          (∃ r′, alive(r′)  ∧  ident(r) > ident(r′)
               ∧  dist(location(r), location(r′)) ≤ D_max)
          ∨  r = companion
```

The Life-Line Problem

Iterative refinement :

- which robot to follow ? identifiers (scout = 0)

The Life-Line Problem

Iterative refinement :

- which robot to follow ? identifiers (scout = 0)
- avoiding collisions robot elimination (alive/dead, defensive)

The Life-Line Problem

Iterative refinement :

- which robot to follow ? — identifiers (scout = 0)
- avoiding collisions — robot elimination (alive/dead, defensive)
- connection despite elimination — communication with lights

What could be a **correct** protocol candidate

1. Chose a target robot (direction)
2. Chose a destination
3. Safe to go ?
   - Yes : go to destination, no warning
   - No : stay, warn of danger

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```
Definition protocole (s : observation) : R2*light :=
 let target := choose_target s in
 let new_pos := choose_new_pos s (fst target) in
 match move_to s new_pos with      (* Is this dangerous? *)
 | true ⇒ (new_pos,false)    (* Safe: move + light off. *)
 | false ⇒ ((0,0), true)  (* Danger: stay + light on.  *)
 end.
```

What could be a **correct** protocol candidate

Constraints on

- Target
- Destination

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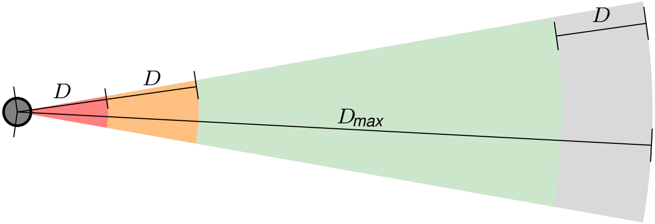
```

What could be a **correct** protocol candidate

```
Axiom choose_new_pos_spec : ∀ obs target,
  let new := choose_new_pos obs target in
    dist new target ≤ Dp
    ∧ dist new (0,0) ≤ D.
```



What could be a **correct** protocol candidate

```
Axiom choose_target_spec : ∀ obs_id local_config,
   let obs := obs_from_config local_config in
   let target := choose_target obs_id obs in
      target ∈ obs                     (* target must be in range *)
    ∧ get_alive target = true                         (* be alive *)
    ∧ get_ident target < get_ident obs_id (* smaller id *)
    ∧ (get_light target = true  (* preferably light off *)
         → ∀ id ∈ obs, get_light id = true)
    ∧ (get_light target = true       (* preferably close *)
         → dist (0,0) (get_loc target) > Dp
         → ∀ id ∈ obs, dist (0,0) (get_loc elt) > Dp).
```

What could be a **correct** protocol family

A family of correct protocols

Theorem. Both invariants hold along any FSYNC execution from legit initial configuration for a candidate fulfilling constraints.

Specifications and proof : 1500 lines of Coq

Solution obtained inside formal framework along specifications

Additional proof : family not empty (easy)

What could be a **correct** protocol family

Actual formal statement of invariants

```
Definition path_conf (cf:config) :=
∀ g, get_alive (cf g) = true →
 get_ident (cf g) = 0
  ∨ ∃ g′, dist (get_loc (cf g)) (get_loc (cf g′)) ≤ Dmax
               ∧ get_alive (cf g′) = true
               ∧ get_launched (cf g′) = true
               ∧ get_ident (cf g′) < get_ident (cf g).
```

```
Definition no_collision_conf(cf:config) := ∀ g g′, g ≠ g′
  → get_launched(cf g) = true → get_launched(cf g′) = true
  → get_alive(cf g) = true → get_alive(cf g′) = true
  → dist (get_loc (cf g)) (get_loc (cf g′)) ≠ 0ℝ.
```

```
Definition NoCollAndPath e :=
forever (fun c ⇒ no_collision_conf c ∧ path_conf c) e.
```

A few words on Pactole

Pactole = formal library for the Coq proof assistant
modelling robotic swarms https://pactole.liris.cnrs.fr/

Assets :

- single framework to express everything
- specification is easy (very close to math)
- proof of correctness + of impossibility
- compare expressive power of models

Examples :

- Space : ring, graph, plane, etc.
- Problems : gathering, convergence, exploration, lifeline